INSTITUTO SUPERIOR TÉCNICO
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Transmission Lines

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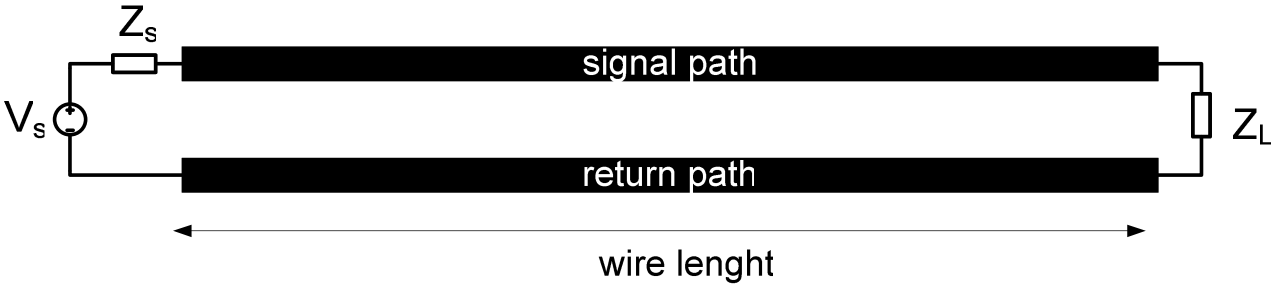
ELINT-MEEC

Outline

- Point to point wiring
- Transmission lines
- Infinite uniform transmission line
- Lossy infinite transmission Lines
- Source and load impedance
- Termination Techniques
- Line impedance and propagation delay

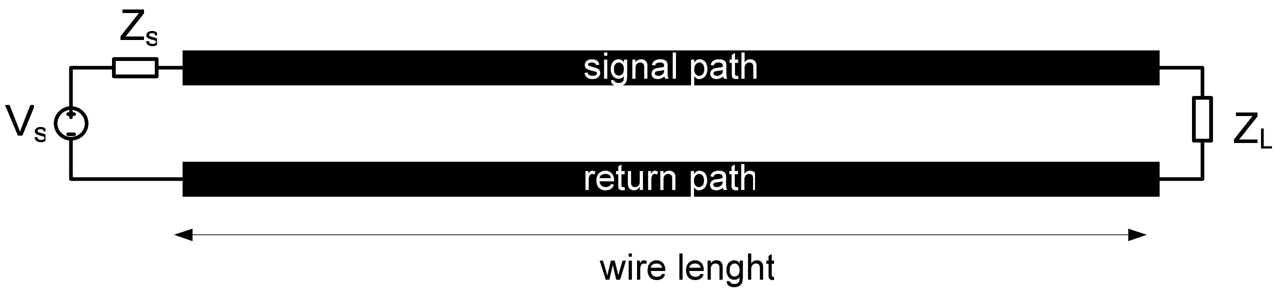
Point to point wiring



Wires:

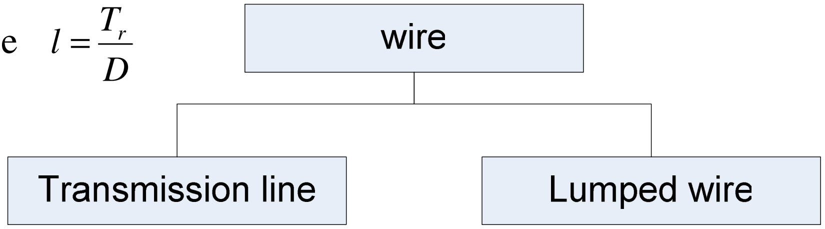
- metal, poly (chip)
- bond wires (package)
- traces (PCB)
- connectors (board to board)
- cables

Point to point wiring



- Lumped versus distributed circuits (transmission lines)

Length of rising edge $l = \frac{T_r}{D}$



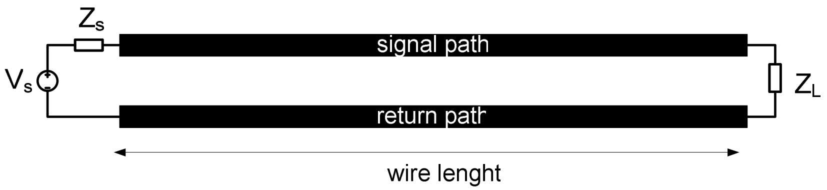
wire lenght > l/6

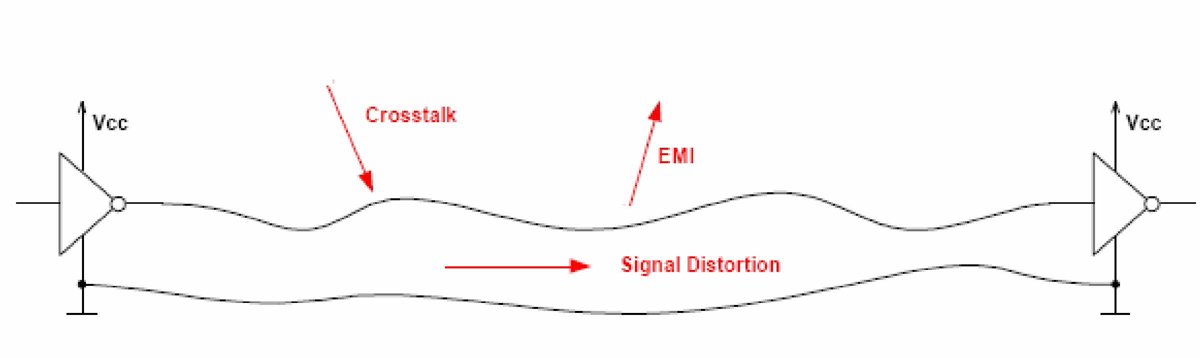
wire lenght < l/6

Point to point wiring

- Transmission lines
  - Disadvantages:
    - Ringing if not terminated
    - More drive power required
  - Advantages:
    - Less distortion
    - Less radiation (EMI)
    - Less crosstalk

Point to point wiring

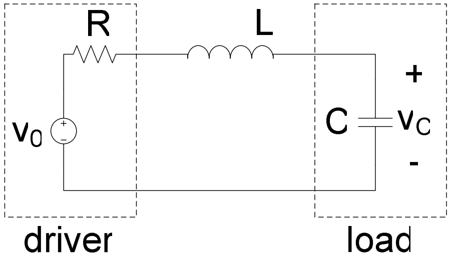




- Problems of ordinary point-to-point wiring (example: wire-wrap prototyping)
    - Signal distortion (due to parasitic components of wires)
    - Radiated noise (EMI – Electromagnetic Interference).
    - Crosstalk

Point to point wiring

- Signal distortion in lumped circuits:



R – driver resistance
C – load capacitance
L – loop inductance

$$H(\omega) = \frac{\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

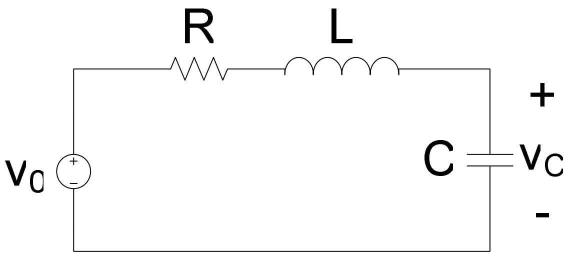
- Step response depends on the Q value

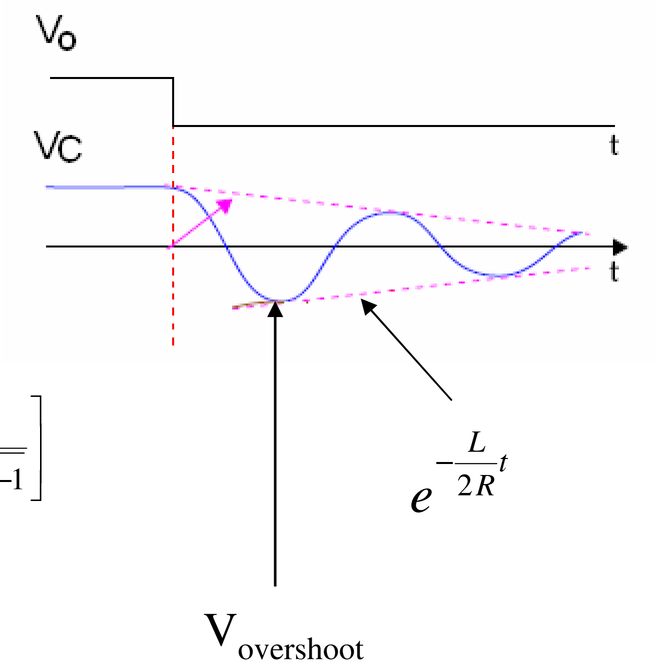
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

  - Q is define as the ratio of energy stored to energy lost per cycle of oscillation.
  - The value of Q indicates how quickly signals on the circuit die out. Low-Q circuits damp quickly while high-Q circuits cause signals to bounce around.

Point to point wiring

- Time domain





$$\text{approximation}: \frac{V_{overshoot}}{V_{step}} = e^{-\left[\frac{\pi}{\sqrt{4Q^2-1}}\right]}$$

  - Q = 1 overshoot 16%
  - Q = 2 overshoot 44%
  - Q < 1/2 no overshoot

Point to point wiring

- Ex:
  - TTL $R_o = 30\ \Omega$ Step 3.7 V
  - L = 89 nH
  - C = 15 pF

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 2.6$$

$$F_{ring} = \frac{1}{2\pi\sqrt{LC}} = 138MHz$$

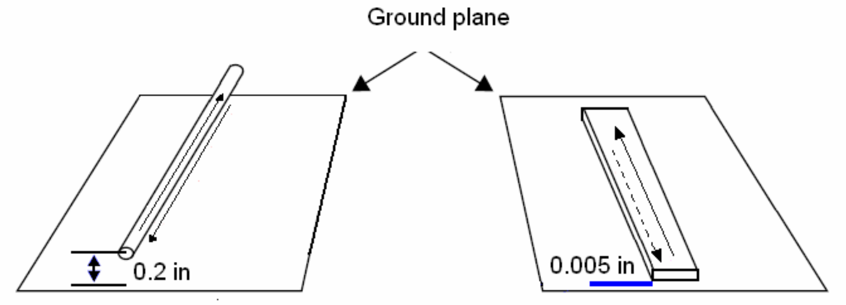
$$V_{\text{overshoot}} = V_{\text{step}} e^{-\left[\frac{\pi}{\sqrt{4Q^2-1}}\right]} = 2V$$

Point to point wiring

- High inductance associated with the point-to-point wiring working into a capacitance load yields a high-Q circuit. Lumped circuits can ring if Q is high.
- Transmission lines, when well terminated, do not ring (wave transmit from source to load and reflected back and forth). Transmission lines reduce ringing (overshoots) by lowering loop inductances.

Point to point wiring

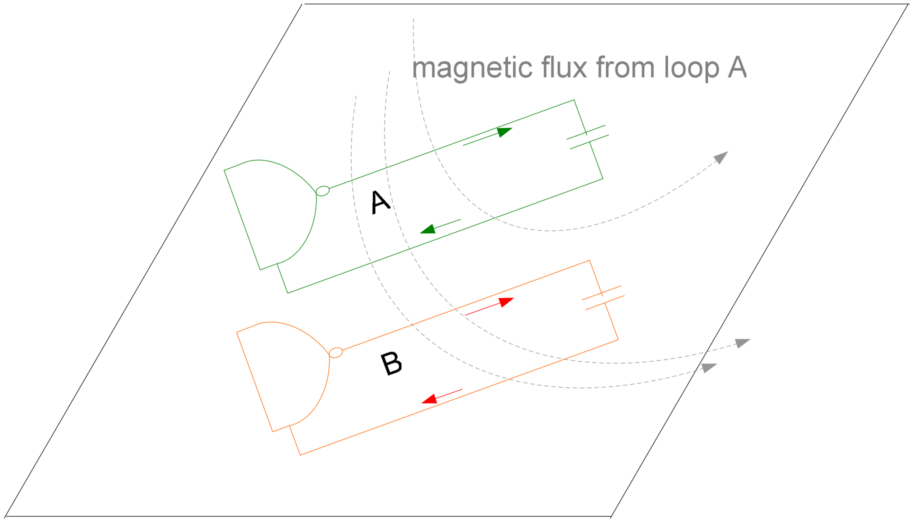
- Electromagnetic Interference (EMI)
  - Magnetic fields generated from fast current loops radiate into space.
  - The magnetic fields are directly proportional to the loop area.



- In ordinary wiring, return current path is at some distance from the signal line, increasing the total current loop area.
- Transmission lines drastically reduce EMI. In transmission lines, the return currents are close to the signal path (magnetic fields cancel).

Point to point wiring

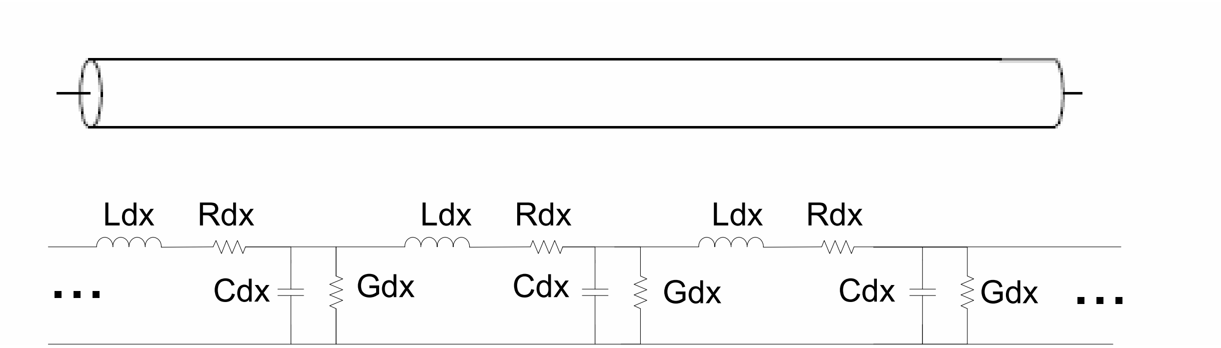
- Crosstalk



- Loops A and B are magnetically coupled (mutual inductance). Loop A fields penetrate loop B. Changes in flux B induces noise in circuit B. Crosstalk arises from changing magnetic fields.
- Transmission lines reduce crosstalk through reduction of loop areas (mutual inductance).

Transmission lines

Long enough wires have to be consider distributed systems



Modeling the transmission line:

- distributed parasitic elements
- parasitic elements of a wire (R, L, C) per unit length
- parasitic elements form an infinitesimal section (cell) of the transmission line
- concatenation of transmission line cells

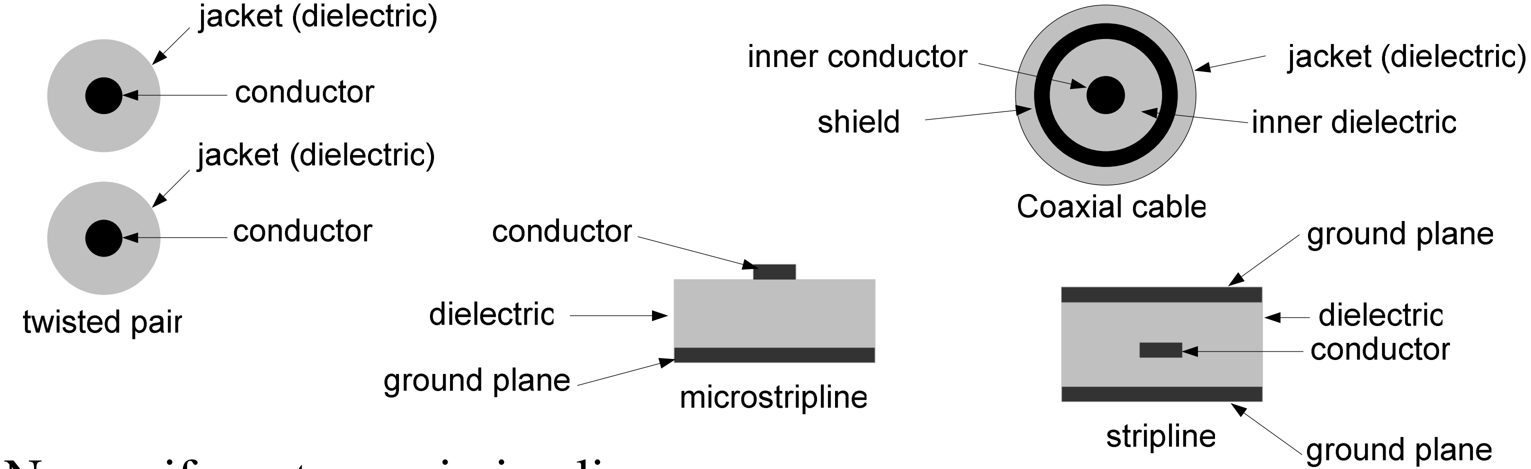
Transmission lines

- Transmission lines characterized by its geometry:
  - cross section: uniform lines or controlled impedance lines if their cross section is the same down the length. Non uniform transmission lines exist when some geometry or material property changes as we move down the length of the line.
  - Similarity of the two conductors: balanced lines with the same shape and size in both conductors. Unbalanced or single ended if the center conductor is much smaller than the outer conductor (return path), usually shared among many signal wires
- Reflections will be minimized and signal quality optimized, if the transmission lines are uniform.
- In general, for most transmission lines, the signal quality and cross-talk effects will be completely unaffected by whether the line is balanced or unbalanced. However, ground-bounce and EMI issues will be strongly affected by the specific geometry of the return path.

Transmission lines

- Uniform transmission lines

  Ex: twisted pair, coaxial cable, microstrip, stripline



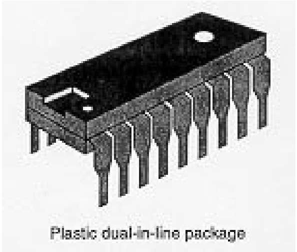
jacket (dielectric)
conductor
jacket (dielectric)
conductor
twisted pair

inner conductor
shield
jacket (dielectric)
inner dielectric
Coaxial cable

conductor
dielectric
ground plane
microstripline

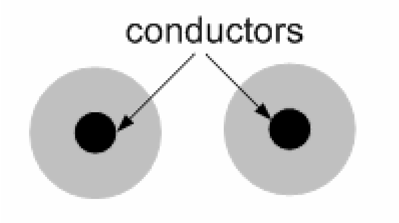
ground plane
dielectric
conductor
ground plane
stripline

- Non uniform transmission lines
  - Ex: pair of leads in a DIP package, usually traces on a PCB without return plane

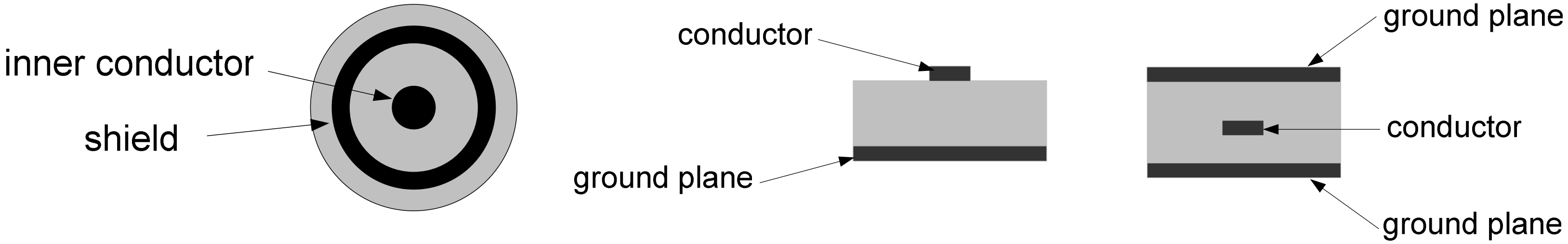


Transmission lines

- Balanced and unbalanced transmission lines:
  - Balanced

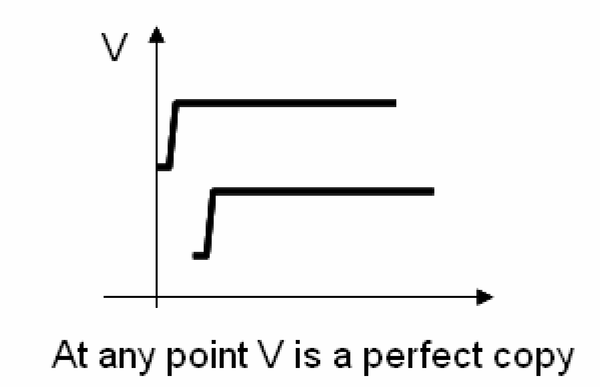


  - Unbalanced or single ended:



Infinite uniform transmission line

- Ideal transmission line (distortionless, lossless):
  - Uniform transmission line with two perfect conductors having zero resistance, and extending forever (the signal starts and goes forever in one direction).
- Properties of ideal transmission lines:
  - signals propagating on line are not distorted
  - signals propagating on line are not attenuated



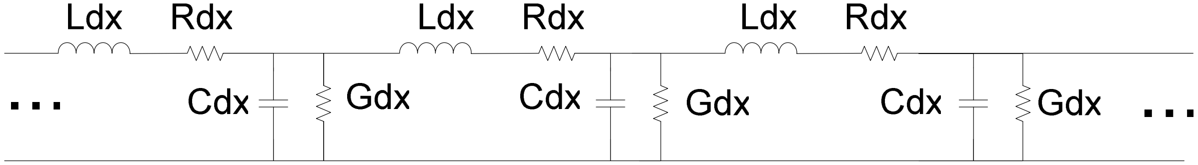
At any point V is a perfect copy

  - Propagation delay: amount of delay per unit length (ps/in)
  - Propagation velocity or transmission velocity (in/ps)

    Light in vacuum: 85 ps/in (100%)

Infinite uniform transmission line

- Propagation delay
  - Related to the series inductance per unit length and parallel capacitance per unit length.

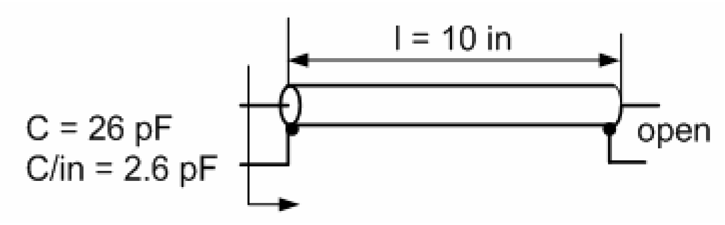


$$propagation\ delay\ (ps/in) = 10^{12}\sqrt{\frac{L}{in}\frac{C}{in}}$$

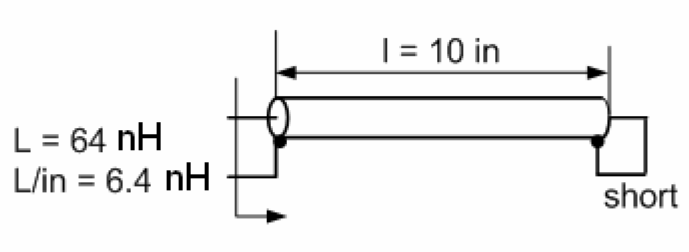
Infinite uniform transmission line

Measuring a coaxial cable (RG-58U)

- Resistance of center conductor (approximately ideal) R = 0.9mΩ/in
- Capacitance: open circuit



- Inductance: short circuit

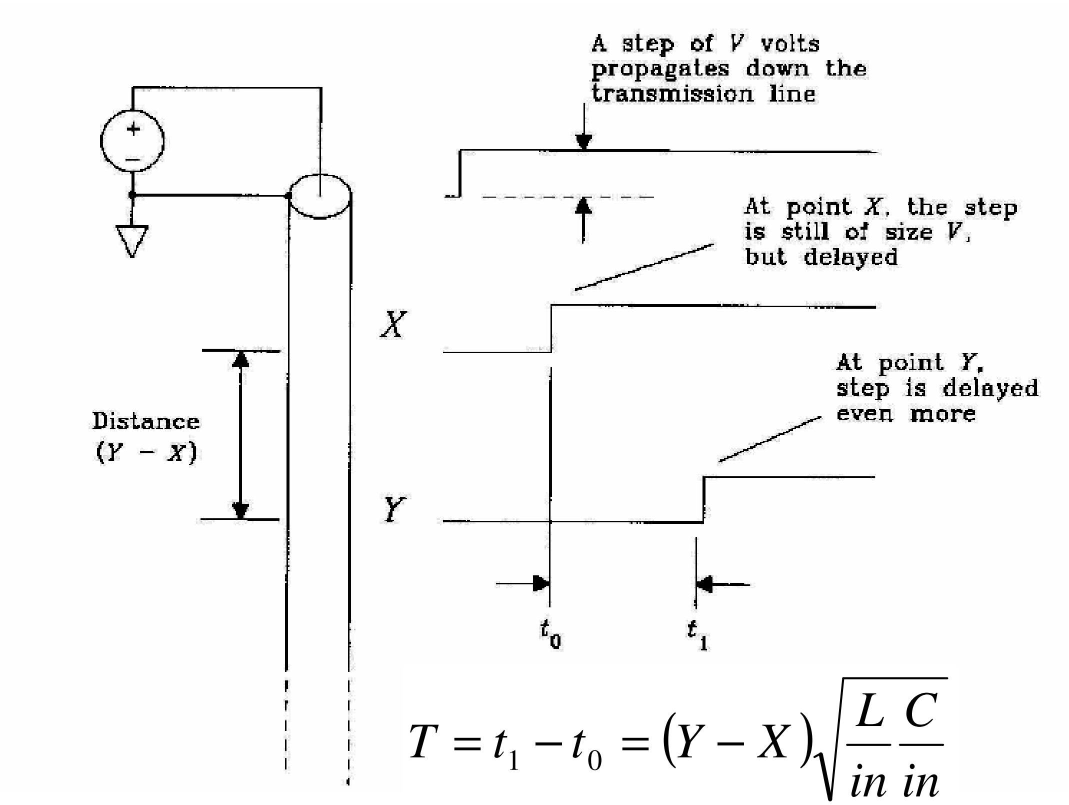


$$propagation\,delay(ps/in) = 10^{12}\sqrt{2.6\times 6.4} = 15.2\,ps/in$$

$$\frac{129}{85} = 4.8\%$$

Infinite uniform transmission line

Determining input impedance Z_0 of a transmission line by impressing a step response


A step of V volts propagates down the transmission line
At point X, the step is still of size V, but delayed
At point Y, step is delayed even more
X
Y
Distance (Y − X)
t_0
t_1

$$T = t_1 - t_0 = (Y - X)\sqrt{\frac{L}{in}\frac{C}{in}}$$

$$Z_0 = \frac{V}{I}$$

$$I = \frac{\text{charge}}{T}$$

$$\text{charge} = C_{XY}V = \frac{C}{in}(Y - X)V$$

$$I = \frac{\frac{C}{in}(Y - X)V}{(Y - X)\sqrt{\frac{L}{in}\frac{C}{in}}} = \sqrt{\frac{C/in}{L/in}}V$$

$$Z = \frac{V}{I} = \sqrt{\frac{L/in}{C/in}} = \sqrt{\frac{L}{C}}$$

Infinite uniform transmission line

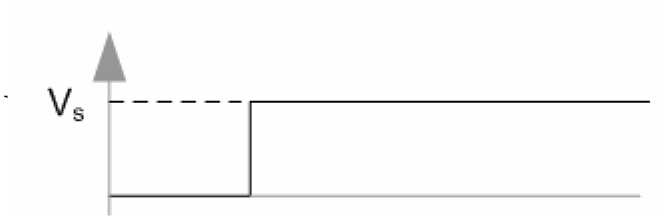
- The characteristic impedance of the line, Z_0, is the instantaneous impedance a signal sees as it propagates down the line.
- In an ideal transmission line, Z_0 is a constant, not depending on the frequency

$$Z_0 = \sqrt{\frac{L}{C}}$$

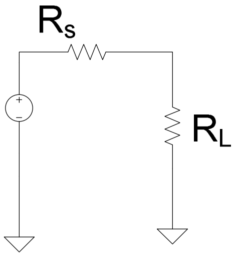
- Examples:
  - Coaxial cable RG174: 50 Ω
  - Coaxial cable RG58: 52 Ω
  - Coaxial cable RG59: 75 Ω
  - Coaxial cable RG62: 93 Ω
  - Twisted pair: 100 a 130 Ω
  - TV antenna: 300 Ω
  - Coaxial cable TV:75 Ω
  - PCB microstrip and stripline used: typically 50 to 75 Ω

Infinite uniform transmission line

Driving a signal

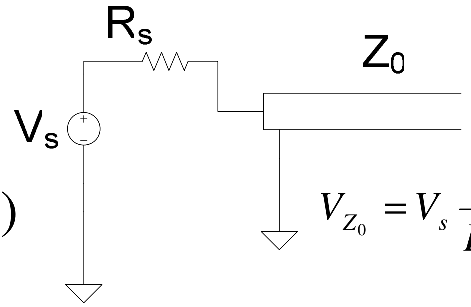


- into a resistor



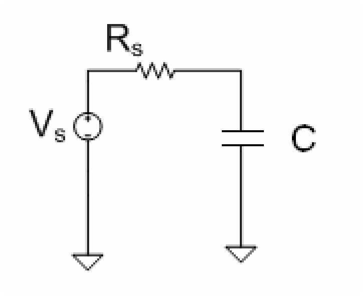
$$V_{R_L} = V_s \frac{R_L}{R_s + R_L}$$

- into a transmission line (during the transmission time, the impedance of line behaves like a resistance equivalent to Z_0)



$$V_{Z_0} = V_s \frac{Z_0}{R_s + Z_0}$$

- into a capacitor

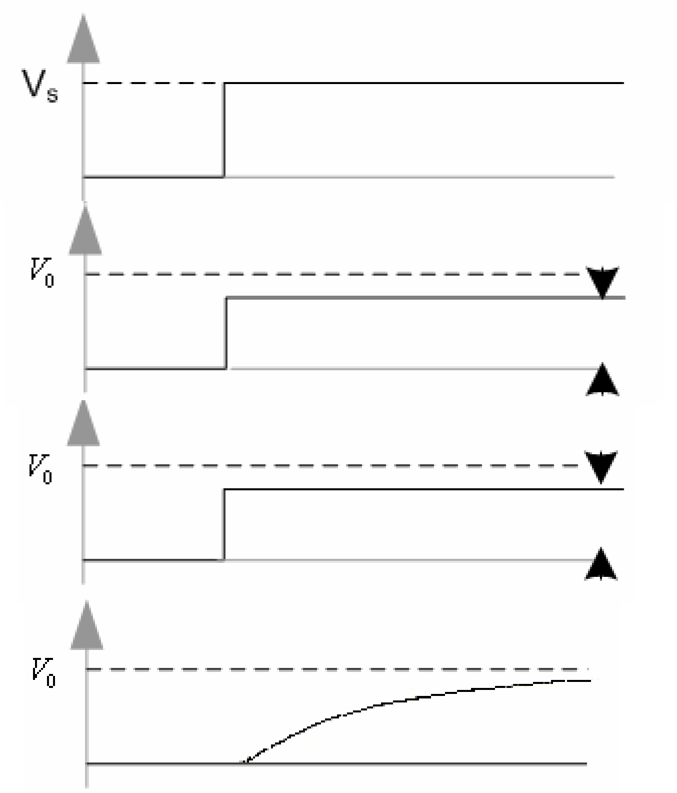


$$V_C = V_s \left(1 - e^{-\frac{t}{\tau}} \right)$$

Infinite uniform transmission line

Driving a signal

- into a resistor
- into a transmission line
- into a capacitor



$$V_{R_L} = V_s \frac{R_L}{R_s + R_L}$$

$$V_{Z_0} = V_s \frac{Z_0}{R_s + Z_0}$$

$$V_C = V_s \left(1 - e^{-\frac{t}{\tau}}\right)$$

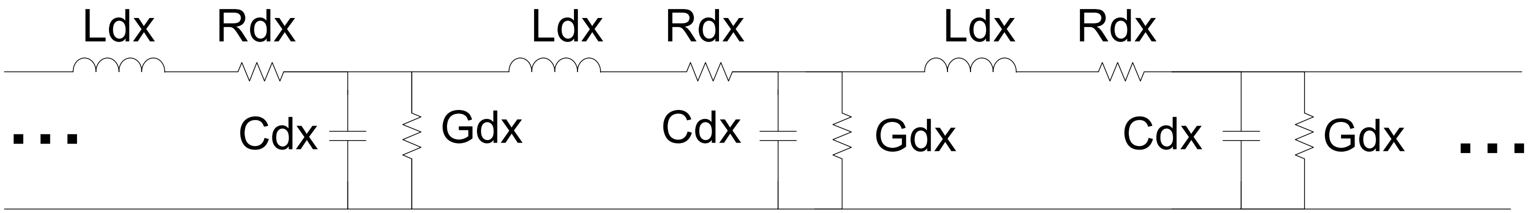
- An ideal transmission line acts exactly like a resistive load. Voltage propagating down the transmission line is a fraction of the drive voltage.

Revising last lecture

- When can an interconnection be considered a lumped or a distributed circuit (transmission lines)?
- Compare: distortion, radiation (EMI), crosstalk in point to point wiring and transmission lines.
- Why there is distortion on point to point wiring ?
- Why radiation and crosstalk are reduced using transmission lines?
- Indicate common transmission lines (cables and connection in a PCB).
- Characterize an ideal transmission line: length, cross section and resistance.
- What is a the equation for the characteristic impedance of a lossless transmission line?
- Characterize signals propagation on ideal transmission lines in terms of distortion and attenuation.

Revising last lecture

- What is the model of a transmission line ?
- What is represented R, L, C and G?



Revising last lecture

- The distance from a home to the power plant is 5000 km. Power line frequency is 50 Hz . Shall the interconnection be considered a transmission line?

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{50} = 6\times10^6 m \qquad d = 5\times10^6 m$$

Revising last lecture

- A signal with a frequency approaching 10 GHz is used on a PCB trace in an FR4 PCB ($e_r = 4$). Shall the interconnection be considered a transmission line?

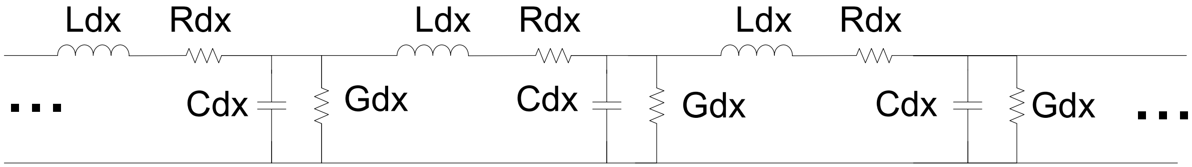
$$\lambda = \frac{v}{f} = \frac{\frac{c}{\sqrt{\varepsilon_r}}}{10\times10^9} = \frac{\frac{3\times10^8}{2}}{10\times10^9} = 1.5\times10^{-2}\,m = 1.5\,cm$$

d cm

$$\frac{l}{6} = \frac{\frac{0.1}{10\times10^9}\times\frac{3\times10^8}{\sqrt{4}}}{6} = 0.025cm$$

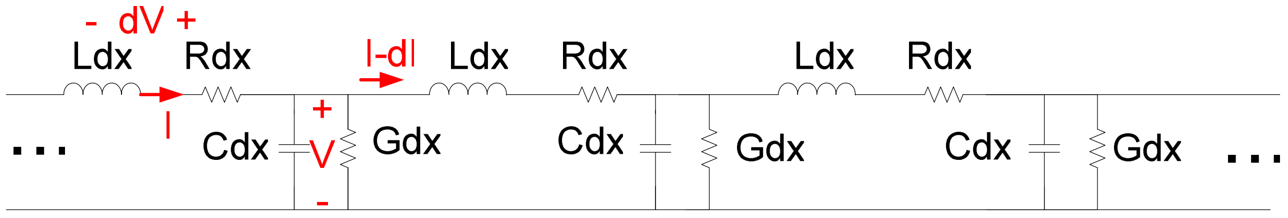
Lossy infinite transmission line

- Resistance in wires (signal and return paths) adds:
  - Attenuation: signal amplitude reduces when traveling along the line
  - Distortion: frequency components of the signal are attenuated and phase shifted by different amounts when traveling along the line



Lossy infinite transmission line

- Time domain equations (differential equations)



$$dV = -R\ I\ dx - L\frac{dI}{dt}dx$$

$$\frac{dV}{dx} = -R\ I - L\frac{dI}{dt}$$

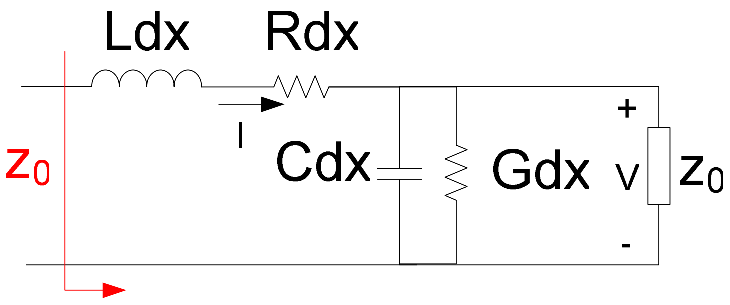
$$dI = -Cdx\frac{dV}{dt} - GV$$

$$\frac{dI}{dx} = -C\frac{dV}{dt} - GV$$

$$\frac{d^2V}{dx^2} = RGV + (RC + LG)\frac{dV}{dt} + LC\frac{d^2V}{dt^2}$$

Lossy infinite transmission line

- Frequency domain: characteristic impedance



$$Z_0 = Rdx + j\omega Ldx + \frac{1}{Gdx + j\omega Cdx + Z_0^{-1}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C + G}}$$

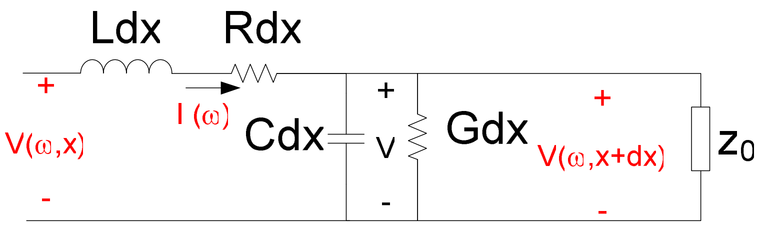
- Characteristic impedance is complex and depend on frequency.
- Whenever R = G = 0 characteristic impedance independent of frequency

$$Z_0 = \sqrt{\frac{L}{C}}$$

(ideal transmission Line)

Lossy infinite transmission line

- Frequency domain equations



$$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C + G}}$$

$$\frac{dV(\omega)}{dx} = -(R + j\omega L)\, I(\omega) = -(R + j\omega L)\frac{V(\omega)}{Z_0}$$

$$\frac{dV(\omega)}{dx} = \sqrt{(j\omega C + G)(R + j\omega L)}\; V(\omega)$$

$$A = \sqrt{(j\omega C + G)(R + j\omega L)}$$

$$\frac{dV(\omega)}{dx} = A\, V(\omega)$$

$$V(\omega, x) = V(\omega, 0)\, e^{-Ax}$$

A - propagation coefficient

Lossy infinite transmission line

- Attenuation and distortion

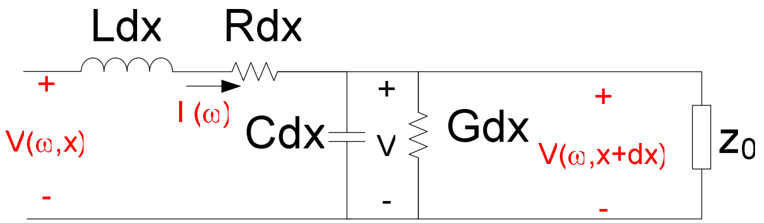
$$V(\omega, x) = V(\omega,0)\, e^{-Ax} \qquad A = \sqrt{(j\omega C + G)(R + j\omega L)}$$

$$H_x(\omega) = \frac{V(\omega, x)}{V(\omega,0)} = e^{-Ax} = e^{-\mathrm{Re}(A)x}\; e^{-\mathrm{Im}(A)x}$$

- Attenuation depend on frequency ω $e^{-x\,\mathrm{Re}\left(\sqrt{(j\omega C+G)(R+j\omega L)}\right)}$
- Phase shift depend on frequency ω $e^{-x\,\mathrm{Im}\left(\sqrt{(j\omega C+G)(R+j\omega L)}\right)}$

Lossy infinite transmission line

- Transfer function $H_x(\omega) = e^{-Ax} = e^{-\sqrt{(G+j\omega C)(R+j\omega L)}\,x}$



- Whenever R = G = 0 $A = 0 + j\sqrt{LC}\,\omega$
  - no attenuation $\operatorname{Re}\left(\sqrt{(G+j\omega C)(R+j\omega L)}\right) = 0$
  - pure delay $\operatorname{Im}\left(\sqrt{(G+j\omega C)(R+j\omega L)}\right) = \omega\sqrt{LC}$

Lossy infinite transmission line

- G models the effects of leakage due to wet or imperfect insulation, between the signal conductors.
- G is effectively zero for most digital transmission line problems below 1 GHz.
- Characteristic impedance (G =0)

$$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}}$$

- At low frequencies (RC line) $R >> \omega L \Rightarrow Z_0$ inversely proportional to $\frac{1}{\sqrt{\omega}}$
- At high frequencies (low loss) $R << \omega L \Rightarrow Z_0 = \sqrt{\frac{\text{L}}{\text{C}}}$ is flat (constant value)
- Practical lines can exhibit both modes

Lossy infinite transmission line

$$H_x(\omega) = e^{-A\,x}$$

$$A = \sqrt{j\omega CR + j\omega C(j\omega L)} = \sqrt{j\omega CR - \omega^2 LC =} j\omega\sqrt{LC}\sqrt{1 - j\frac{R}{\omega L}}$$

$$\omega >> \frac{R}{L} \qquad \Leftrightarrow \qquad \frac{R}{\omega L} << 1 \qquad A \approx j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right) = \frac{1}{2}R\sqrt{\frac{C}{L}} + j\sqrt{LC}\,\omega$$

$$A = j\omega\sqrt{LC}\sqrt{1 - jx} \qquad x = \frac{R}{\omega L}$$

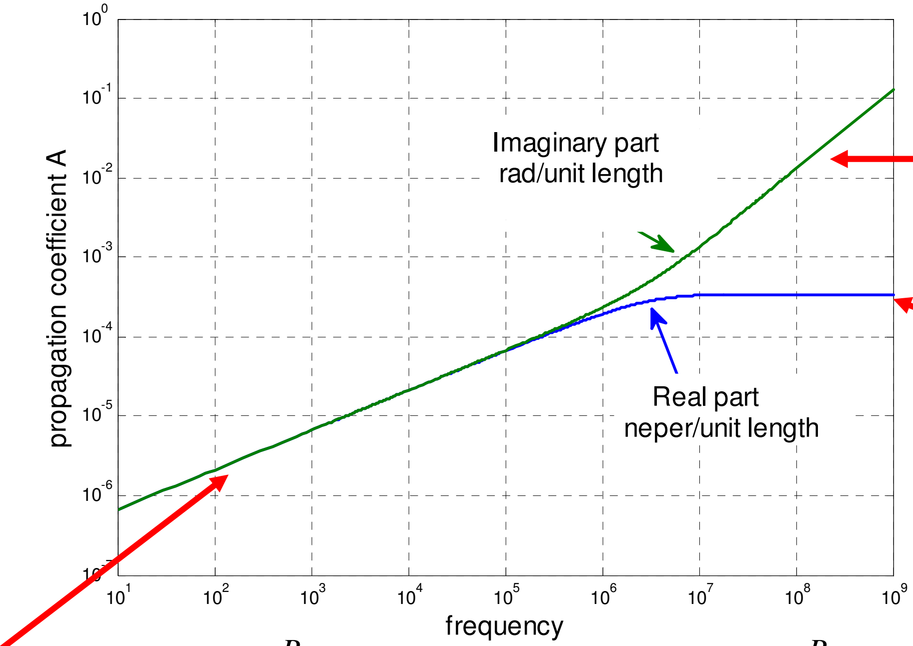
$$|x| << 1 \Rightarrow \sqrt{1 - jx} \approx 1 - \frac{1}{2}jx$$

$$\omega << \frac{R}{L} \qquad \Leftrightarrow \qquad \frac{R}{\omega L} >> 1$$

$$A \approx j\omega\sqrt{LC}\sqrt{-j\frac{R}{\omega L}} = \sqrt{j\omega CR} == \frac{1}{\sqrt{2}}(1 + j)\sqrt{\omega CR}$$

Lossy infinite transmission line

Ex: RG-58U coaxial cable



Im(A) linear

Re(A) constant

Im(A),Re(A)

$\sqrt{\omega}$

$\omega << \frac{R}{L}$

$\omega >> \frac{R}{L}$

$$A \approx \sqrt{j\omega CR}$$

$$\mathrm{Re}(A), \mathrm{Im}(A) \propto \sqrt{\omega}$$

$$\mathrm{Re}(A) = \frac{1}{2} R \sqrt{\frac{C}{L}}$$

$$\mathrm{Im}(A) = j\sqrt{LC}\,\omega$$

RC transmission line

$$\omega << \frac{R}{L} \Rightarrow A \approx \sqrt{j\omega RC}$$

- RC transmission line (also called diffusion line)
  - arg(A) = 45°
    - both the real part (log of attenuation) and imaginary part (phase in radians) are proportional to the square root of the frequency
  - Small attenuation
  - Distortion
  - Restricts signal frequencies to less than R/L: F_{knee} needs to be smaller than R/L which is rarely the case in high speed digital circuits.

Low loss transmission line

$$\omega >> \frac{R}{L} \Rightarrow \mathrm{A} = \frac{1}{2} R \sqrt{\frac{C}{L}} + j\sqrt{LC}\,\omega$$

- Low loss transmission line
  - Attenuation is flat
  - Phase shift increases linearly.
  - High importance in high speed digital circuits

Characteristic impedance	$Z_0 = \sqrt{L/C}$
Attenuation at X length	$e^{-\frac{R}{2\sqrt{L/C}}X}$
Loss per unit length	$4.34\left[R/\sqrt{L/C}\right]_{dB}$
Delay per unit length	$T_p = \sqrt{LC}$

Lecture exercise

- Show that loss per unit length (dB) in a low loss transmission line

$$4.34\left[\frac{R}{\sqrt{L/C}}\right]_{dB}$$

Attenuation in a low loss transmission: $e^{-\frac{R}{2\sqrt{L/C}}x}$

Attenuation in a low loss transmission per unit length: $e^{-\frac{R}{2\sqrt{L/C}}}$

Attenuation in a low loss transmission per unit length in dBs

$$20\log 10\left(e^{\frac{1}{2}R\sqrt{\frac{C}{L}}}\right)=20\times\frac{\ln(e^{\frac{1}{2}R\sqrt{\frac{C}{L}}})}{\ln 10}=20\frac{1}{2{,}303}\frac{1}{2}R\sqrt{\frac{C}{L}}=4.34R\sqrt{\frac{C}{L}}$$

Lecture exercise

- A digital signal is transmitted by a transmission line with 50 Ω of characteristic impedance. Nominal loss value is 0.2dB. Calculate how much resistance is tolerable?

$$4.34\frac{R}{\sqrt{L/C}}x = 0.2$$

$$4.34\frac{R}{Z_0}x = 0.2$$

$$Rx = 0.046 \times Z_0$$

$$Rx = 0.046 \times 50 = 0.23\ \Omega$$

Lecture exercise

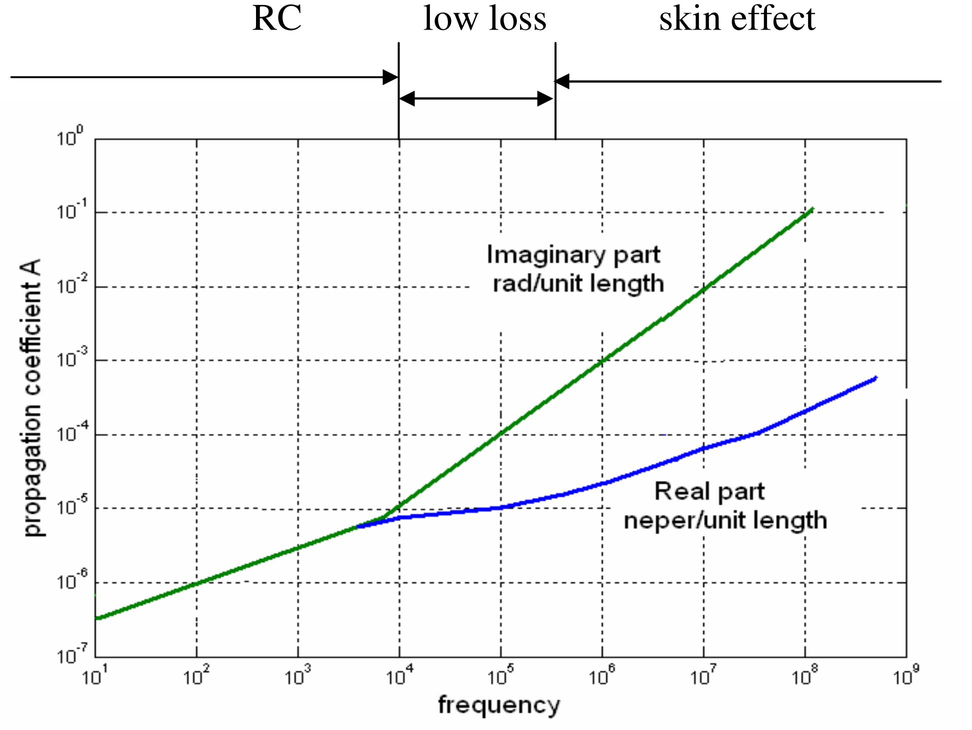
- Consider a phone line using two AWG 24 wires. The wires are twisted yielding R= 0.0042 Ω /in, L=10nH/in and C=1pF/in. Being the center frequency of the voice band on telephones lines 1.6 kHz, justify the use of a termination value of 600 Ω.

$$Z_L = Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}}$$

$$Z_0 = \sqrt{\frac{0.0042 + j2\pi \times 1600 \times 10n}{j2\pi \times 1600 \times 1p}} = 648\,\Omega\angle -45^{\circ}$$

Skin effect

- Series resistance of transmission lines also depend on frequency.



RG-58/U

Series resistance begins going up.

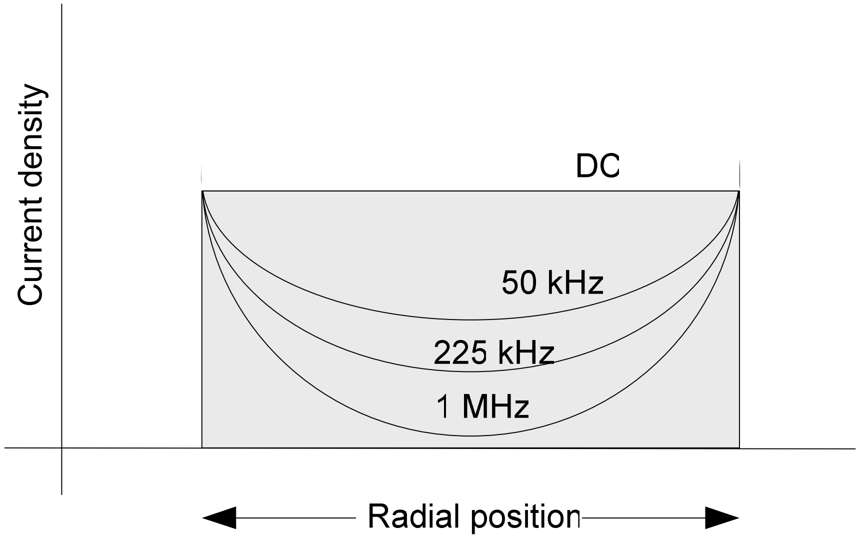
$$\mathrm{Re}(A) = \frac{1}{2}\enclose{circle}{R}\sqrt{\frac{C}{L}}$$

$$\mathrm{Im}(A) = j\sqrt{LC}\,\omega$$

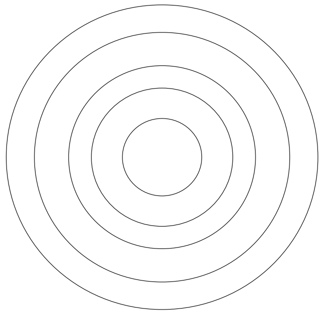
This causes more attenuation (more loss) but maintains linear phase.

Skin effect

- At low frequencies, the distribution of current density inside a conductor is uniform
- At high frequencies, the current density grows larger near the surface of the wire and almost no current flows in the center.
- At high frequencies, current pushes out toward the surface.



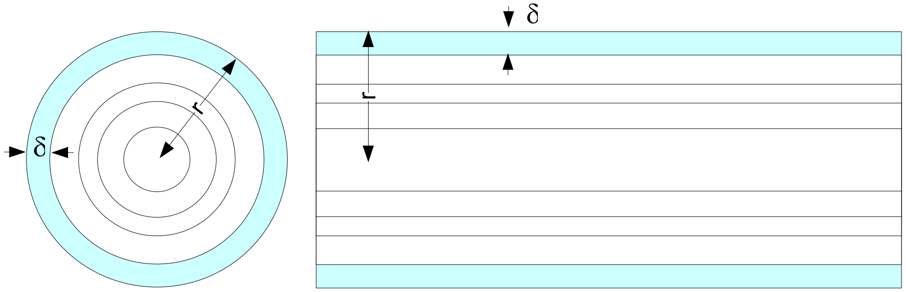
Current follows the path of least inductance.



Inductance of inner rings is higher than outer rings
Mutual inductance between rings forces current against the outer surface

Skin effect

- Skin depth (δ): the distribution of current penetration falls off exponentially from outer to inner shell.
- Skin depth is inversely proportional to the square root of frequency.



$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

- Conducting area $Area = r^2\pi - (r-\delta)^2\pi = \pi\left(2r\delta - \delta^2\right)$

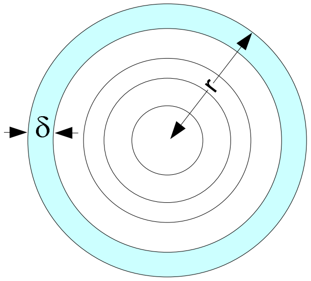
- For high frequencies (δ << r): $Area \approx \pi 2r\delta$

Skin effect

- AC resistance (high frequency) is inversely proportional to the skin depth. Therefore it is directly proportional to the square root of the frequency:

$$R_{AC}(\text{unit length}) \approx \frac{\rho}{\pi 2r\delta} = \frac{\rho}{\pi 2r\sqrt{\frac{2\rho}{\omega\mu}}} = \frac{\sqrt{\pi f \mu\rho}}{2r\pi}$$

- Here, R_{AC} takes over when the skin depth is less than the thickness of the conductor:
  - for round conductors the radius conductor
  - for rectangular conductors (PCB traces) half the conductor thickness



- DC resistance is finite. An approximation that combines DC and AC resistance elements:

$$\mathrm{R}(f) = \sqrt{R_{DC}^2 + \mathrm{R}_{\mathrm{AC}}^2(f)}$$

Skin effect

Skin region:

$$R_{AC} \approx \frac{\sqrt{\pi f \mu \rho}}{2 r \pi}$$

$$\mathrm{R}(f) = \sqrt{R_{DC}^2 + \mathrm{R}_{\mathrm{AC}}^2(f)}$$

$$e^{-\frac{R(f)}{2\sqrt{\frac{L}{C}}}x}$$

- Attenuation (per unit length in dB) proportional to the square root of frequency
- As total attenuation is proportional to the length, cutting the cable in half improves the frequency response by a factor of 4 (reduces the attenuation by a factor of 2).

Skin effect

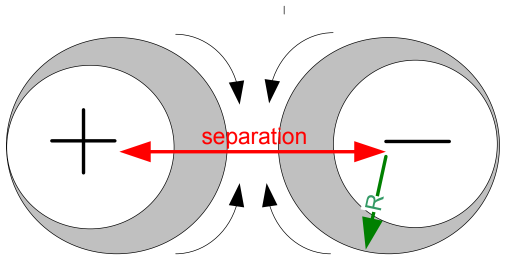
Skin region:

$$Z_0 = \sqrt{\frac{R(f) + j\omega L}{j\omega C}} \qquad R_{AC} = c^{te}\sqrt{\omega}$$

- the ωL term grows linearly with ω, while the $R_{AC}(\omega)$ term grows $\omega^{1/2}$. The characteristic impedance is not much affected by the skin effect.

Proximity Effect

- Current distribution in a conductor is affected by currents in adjacent conductors.
- Opposing currents in adjacent wires, increase current density distribution at the inside surface of the two wires, caused by changing magnetic fields.



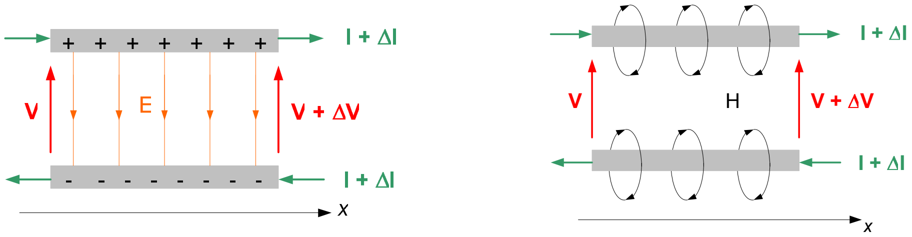
- Proximity effect does not occur for low frequencies.
- Proximity effect dependents on the ratio of wire separation between centers to wire diameter

Proximity Effect

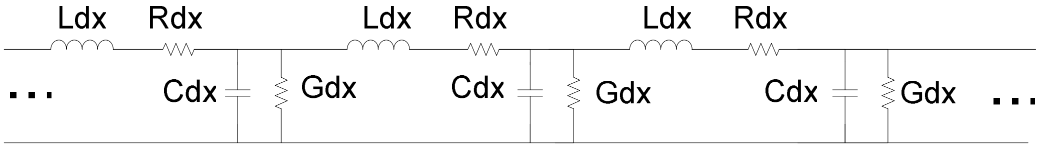
- Proximity effect is responsible for a larger effective resistance at high frequencies (like skin effect).
- Proximity effect does not increase with frequency (unlike skin effect). Proximity effect is in equilibrium at rather low frequency than skin effect frequency
- Proximity effect causes the return current in a ground plane to follow closely underneath the signal conductor. Current flows in a formation that minimizes the total loop impedance.

Revising last lecture

- When does an interconnection be considered a transmission line?



- In a transmission line, what represent R and G?



- What are G and R values for an ideal transmission line?
- What is approximately G value for frequencies bellow 1 GHz?

Revising last lecture

- When can an infinite transmission line be considered RC or low loss transmission line?

$$Z_0 = \sqrt{\frac{R + j\omega L}{0 + j\omega C}} \qquad \omega << \frac{L}{R} \Rightarrow Z_0 = \sqrt{\frac{R}{j\omega C}} \qquad \omega >> \frac{L}{R} \Rightarrow Z_0 = \sqrt{\frac{\mathrm{L}}{\mathrm{C}}}$$

- How is attenuation and distortion in an RC transmission line?

$$H_x(\omega) = e^{-Ax} = e^{-\alpha x} e^{-j\beta x} \qquad A \approx \sqrt{j\omega CR} = \frac{1}{\sqrt{2}}(1 + j)\sqrt{\omega CR}$$

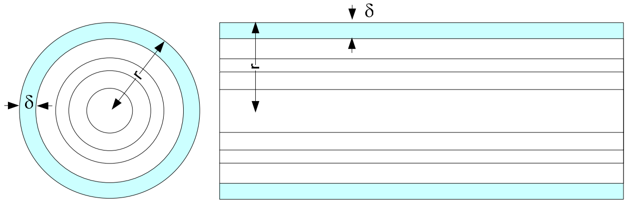
- How is attenuation and distortion in a low loss transmission line? How do attenuation and distortion depend on frequency?

$$H_x(\omega) = e^{-Ax} \qquad A \approx \frac{1}{2} R\sqrt{\frac{C}{L}} + j\sqrt{LC}\omega$$

- In high speed digital circuits shall we consider RC or low loss transmission lines?

Revising last lecture

- What is the skin effect ?



$$R_{AC}\,(\text{unit length}) \approx \frac{\sqrt{\pi\, f \mu \rho}}{2 r \pi}$$

- Does the skin effect limit the frequency response of infinite transmission lines?
- How is the variation of attenuation in dBs due to the skin effect for high frequencies in transmission lines?

attenuation proportional to square root of frequency

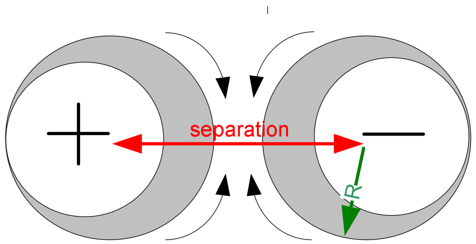
Revising last lecture

- How does the skin effect affect characteristic impedance?

  not much

$$Z_0=\sqrt{\frac{R(f)+j\omega L}{j\omega C}} \longrightarrow \sqrt{\omega}$$

- What is the proximity effect?



  - larger effective resistance at high frequencies (like skin effect).
  - is in equilibrium at rather low frequency than skin effect frequency

Exercise

A square wave is launched at the beginning of a low loss transmission line

a) Determine the wave form at length l.

b) Determine the attenuation and distortion.

$$H(\omega) = e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}\,l - j\sqrt{LC}\,\omega\, l}$$

$$v_{x=0}(t) = \frac{4}{\pi}\left(\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t + \cdots\right)$$

ω 3ω 5ω 7ω

$$e^{j\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l + j\omega\left(t - \sqrt{LC}l\right)}$$

$$e^{j3\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l + j3\omega\left(t - \sqrt{LC}l\right)}$$

$$e^{j5\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l + j5\omega\left(t - \sqrt{LC}l\right)}$$

$$e^{j7\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l + j7\omega\left(t\sqrt{LC}l\right)}$$

Exercise

$$\begin{tikzcd} e^{j\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l+j\omega\left(t-\sqrt{LC}l\right)} \arrow[d] \\ \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos\omega\left(t-l\sqrt{LC}\right) \end{tikzcd}$$

$$\begin{tikzcd} e^{j3\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l+j3\omega\left(t-\sqrt{LC}l\right)} \arrow[d] \\ \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 3\omega\left(t-l\sqrt{LC}\right) \end{tikzcd}$$

$$\begin{tikzcd} e^{j5\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l+j5\omega\left(t-\sqrt{LC}l\right)} \arrow[d] \\ \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 5\omega\left(t-l\sqrt{LC}\right) \end{tikzcd}$$

$$\begin{tikzcd} e^{j7\omega t} \rightarrow e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l+j7\omega\left(t-\sqrt{LC}l\right)} \arrow[d] \\ \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 7\omega\left(t-l\sqrt{LC}\right) \end{tikzcd}$$

Exercise

$$\frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos\omega\left(t-l\sqrt{LC}\right) \qquad \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 3\omega\left(t-l\sqrt{LC}\right) \qquad \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 5\omega\left(t-l\sqrt{LC}\right) \qquad \frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\cos 7\omega\left(t-l\sqrt{LC}\right)$$

$$v_{x=l}(t)=\frac{4}{\pi}e^{-\frac{1}{2}R\sqrt{\frac{C}{L}}l}\left(\cos\omega\left(t-l\sqrt{LC}\right)-\frac{1}{3}\cos 3\omega\left(t-l\sqrt{LC}\right)+\frac{1}{5}\cos 5\omega\left(t-l\sqrt{LC}\right)-\frac{1}{7}\cos 7\omega\left(t-l\sqrt{LC}\right)+\cdots\right)$$

attenuation

delay

Dielectric loss

G (dielectric losses)

- The heat absorbed by a dielectric material in the presence of changing electric fields is proportional to the dielectric loss factor for that material.
- Dielectric loss in transmission lines translates into signal attenuation.
- Dielectric loss is a function of frequency.
- Loss in FR-4 (glass epoxy) is negligible for digital apps below 1 GHz. In analog applications loss problem exist for high Q circuits
- Ceramic substrates, alumina, are used for apps with $f > 1$ GHz.
- In long cables, the dielectric properties are more significant (at 10 MHz telephone wire has measurable dielectric losses).
- Dielectric losses are often lumped with the skin effect into an overall model.